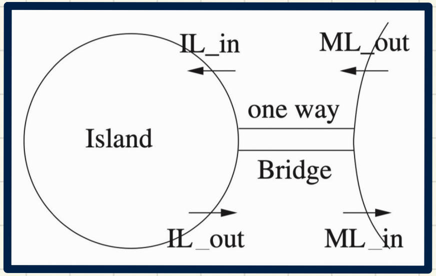
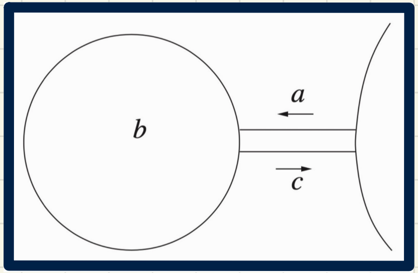
Bridge Controller: Guarded Actions of "new" Events in 1st Refinement





constants: d

axioms:
axm0_1 : $d \in \mathbb{N}$
axm0_2 : $d > 0$

variables: a, b, c

invariants:
inv1_1 : $a \in \mathbb{N}$
inv1_2 : $b \in \mathbb{N}$
inv1_3 : $c \in \mathbb{N}$
inv1_4 : $a + b + c = n$
inv1_5 : $a = 0 \lor c = 0$

IL_in: A car enters **island** (getting off the **bridge**).

```
IL_in
  when
    ??
  then
    ??
  end
```

IL_out: A car exits **island** (getting on the **bridge**).

```
IL_out
  when
    ??
  then
    ??
  end
```

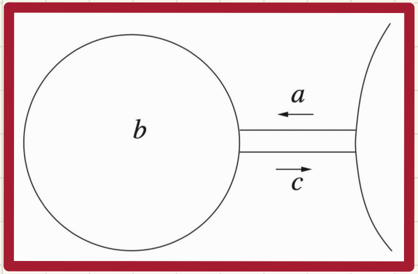
Before-After Predicates of Event Actions: 1st Refinement

```
IL_in
  when
    a > 0
  then
    a := a − 1
    b := b + 1
  end
```

```
IL_out
  when
    b > 0
    a = 0
  then
    b := b − 1
    c := c + 1
  end
```

- Pre-State
- Post-State
- Sate Transition

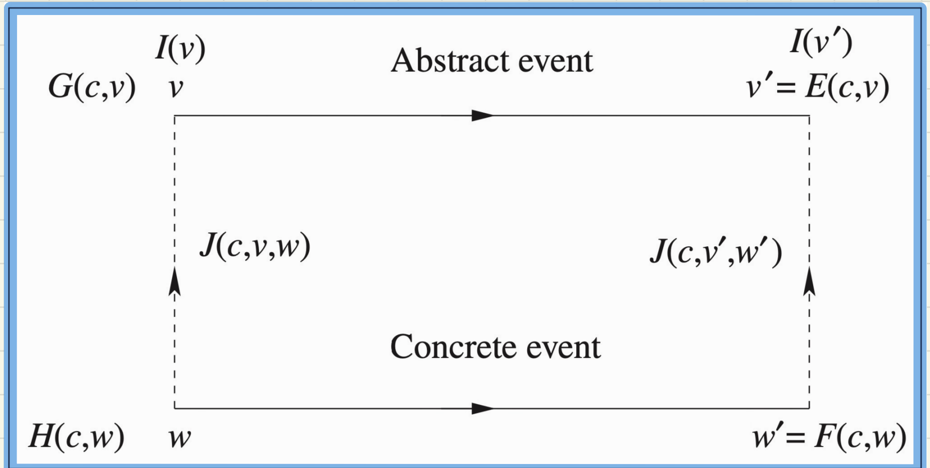
Concrete State Space

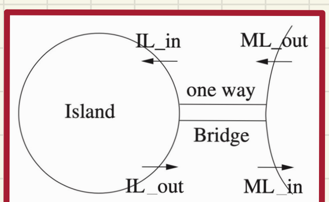


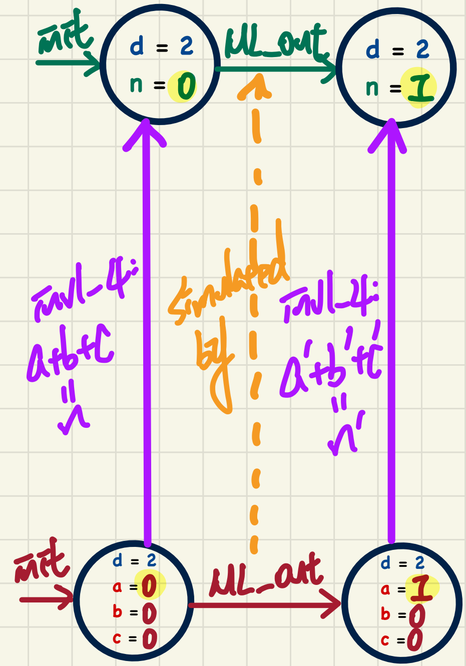
Visualizing Invariant Preservation in Refinement

Each **new** **state transition** (from w to w′)
should be simulated by
an **abstract** **dummy state transition** (from v to v′)

skip
begin
end







PO/VC Rule of Invariant Preservation: Sequents

Abstract m0

IL_in/INV1_4/INV

constants: d

axioms:
axm0_1 : $d \in \mathbb{N}$
axm0_2 : $d > 0$

variables: n

invariants:
inv0_1 : $n \in \mathbb{N}$
inv0_2 : $n \leq d$

$A(c)$
$I(c, v)$
$J(c, v, w)$
$H(c, w)$
$\vdash$
$J_i(c, E(c, v), F(c, w))$

Concrete m1

IL_in/INV1_5/INV

variables: a, b, c

invariants:
inv1_1 : $a \in \mathbb{N}$
inv1_2 : $b \in \mathbb{N}$
inv1_3 : $c \in \mathbb{N}$
inv1_4 : $a + b + c = n$
inv1_5 : $a = 0 \lor c = 0$

```
IL_in
  when
    a > 0
  then
    a := a − 1
    b := b + 1
  end
```

```
IL_out
  when
    b > 0
    a = 0
  then
    b := b − 1
    c := c + 1
  end
```

Q. How many PO/VC rules for model m1?

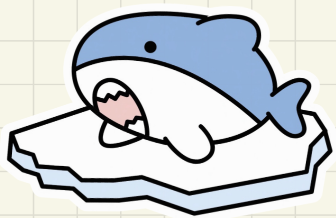
Discharging POs of m1: Invariant Preservation in Refinement

IL_in/inv1_4/INV

$$\frac{H1 \vdash G}{H1, H2 \vdash G} \quad \textbf{MON}$$

$$\frac{}{H, P \vdash P} \quad \textbf{HYP}$$

$$
\begin{array}{l}
d \in \mathbb{N} \\
d > 0 \\
n \in \mathbb{N} \\
n \leq d \\
a \in \mathbb{N} \\
b \in \mathbb{N} \\
c \in \mathbb{N} \\
a + b + c = n \\
a = 0 \lor c = 0 \\
a > 0 \\
\vdash \\
(a - 1) + (b + 1) + c = n
\end{array}
$$



Discharging POs of m1: Invariant Preservation in Refinement

ML_in/inv1_5/INV

$$\frac{}{\bot \vdash P}\ \textbf{FALSE\_L}$$

$$\frac{H1 \vdash G}{H1, H2 \vdash G}\ \textbf{MON}$$

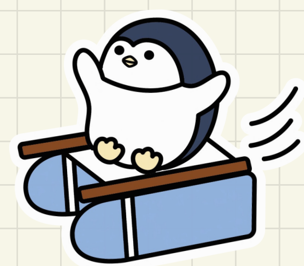
$$\frac{H \vdash Q}{H \vdash P \lor Q}\ \textbf{OR\_R2}$$

$$\frac{}{H, P \vdash P}\ \textbf{HYP}$$

$$\frac{H(F), E = F \vdash P(F)}{H(E), E = F \vdash P(E)}\ \textbf{EQ\_LR}$$

$$\frac{H, P \vdash R \qquad H, Q \vdash R}{H, P \lor Q \vdash R}\ \textbf{OR\_L}$$

$d \in \mathbb{N}$
$d > 0$
$n \in \mathbb{N}$
$n \leq d$
$a \in \mathbb{N}$
$b \in \mathbb{N}$
$c \in \mathbb{N}$
$a + b + c = n$
$a = 0 \lor c = 0$
$a > 0$
$\vdash$
$(a - 1) = 0 \lor c = 0$



Livelock Caused by New Events Diverging

An alternative m1 (for demonstration)

constants: d

axioms:
axm0_1 : $d \in \mathbb{N}$
axm0_2 : $d > 0$

variables: a, b, c

invariants:
inv1_1 : $a \in \mathbb{Z}$
inv1_2 : $b \in \mathbb{Z}$
inv1_3 : $c \in \mathbb{Z}$

ML_out
when
$a + b < d$
$c = 0$
then
$a := a + 1$
end

ML_in
when
$c > 0$
then
$c := c - 1$
end

IL_in
begin
$a := a - 1$
$b := b + 1$
end

IL_out
begin
$b := b - 1$
$c := c + 1$
end